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Tamás MIHÁLYDEÁK

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Cracow University of Economics
Faculty of Economics and International Relations
Rakowicka 27, 31-510 Krakow, Poland
phone +48 12 293 5327, 293 5508
fax +48 12 293 5042, 293 5037
e-mail: papers@uek.krakow.pl; stanekp@uek.krakow.pl; wachk@uek.krakow.pl
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General logical systems of functor–argument decomposition

Tamás Mihálydeák

Department of Computer Science,
Faculty of Informatics, University of Debrecen
Hungary H-4010 Debrecen, P.O. Box 12
mihalydeak.tamas@inf.unideb.hu

Abstract. We consider general logical systems of functorargument decomposition. The defined notion of contexts as introduced here plays a crucial role in defining central logical notions such as satisfiability, consequence relations and validity. We outline the most important possibilities which in turn lead to different logical systems.

Keywords: Context, extensionality, intensionality, intensional logic, partial logic

1 Personal foreword

I am greatly indebted to professor Imre Ruzsa for the opportunity to work with him for almost two decades. After graduation I began to work as a research assistant at Kossuth University, Debrecen in 1979 and I wrote a letter to professor Imre Ruzsa. In spite of the fact that we had never met and did not know each other personally, he answered soon. The first personal meeting changed my scientific life crucially. I have no opportunity to tell the whole story, but I should like to emphasize that I should quote his books and papers[1] in almost each sentence of the present paper which is dedicated to memory of professor Imre Ruzsa.

2 Introduction

From the theoretical point of view, these systems represent function abstraction and function application and rely on functor–argument decomposition, which goes back to Frege.[2]

In Frege's view, one of the most important inventions of Begriffsschrift is the replacement of the subject–predicate decomposition by the functor–argument one. He wrote the following: "The very invention of this Begriffsschrift, it seems

[1] I mention here only three of them:[Ruzsa, 1989], [Ruzsa, 1991], [Ruzsa, 1997].

[2] I use the expression 'functor' instead of 'function' in order to differentiate an incomplete expression of a language from its semantic value.

to me, has advanced logic. ... [L]ogic hitherto has always followed ordinary language and grammar too closely. In particular, I believe that the replacement of the concept subject and predicate by argument and function will prove itself in the long run. It is easy to see how taking a content as a function of an argument gives rise to concept formation. ... The distinction between subject and predicate finds no place in my representation of a judgement."[3] [Frege, 1879, pp. 51, 53]

One of the most general theoretical representations of the functor–argument decomposition is the well–known type theory (or the different systems of type–theoretical language and/or logic[4]).

Generally, syntactic categories have to be distinguished from semantic ones. At the same time our formal systems fulfill the following fundamental principle of formal type–theoretical semantics:

> *The mirror principle*: "Associated with every syntactic category C is a counterpart semantic category C^*, whose *mathematical type* 'mirrors' the *grammatical type* of C. And, in particular, every expression of syntactic category C is interpreted by an object of semantic category C^*." [Dunn at al., 2001, p. 142]

On the basis of the mirror principle in what follows we are speaking about types, and using them to define and denote different syntactic categories and the corresponding sets of possible semantic values.

The proofs of theorems in Section 2,3,4 can be found in [Mihálydeák, 2010, pp. 127–131].

3 General formal system

At first the system of types has to be defined. The system of types relies on primitive type(s). Generally we have only one requirement: the symbol o must be a primitive type. From the theoretical point of view the main reason for this is that the symbol o is taken as the type of the most fundamental expressions of our formal language. Expressions of type o are called formulae. Formulae directly correspond to a special sort of conceptual content or information. It means that formulae are the structures of complete information or closed (and whole) conceptual content. In a given interpretation formulae are intended to have complete information called proposition in the literature.

There is another mainly semantic reason for type o having been declared to be primitive. From the semantic point of view Frege's context principle or as W. Hodges says [Hodges, 2001a], Frege's Dictum can be taken as a general leading idea. In *The Foundation of Arithmetic* Frege wrote the following usually quoted as the context principle:

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[3] I use the expression 'functor' instead of 'function' in order to differentiate an incomplete expression of a language from its semantic value.

[4] It goes back to Church [Church, 1940].

> "never to ask for the meaning of a word in isolation, but only in the context of a proposition;" [Frege, 1884, p. x]
> "It is enough if the proposition taken as a whole has sense; it is this that confers on its parts also their content." [Frege, 1884, p. 71]

According to context principle an expression has sense (meaning) only in the sentence in which it occurs. Sometimes we need more than one primitive type (usually individual names constitute another primitive type). The main difference between primitive and non-primitive types is that the semantic domains of primitive types have to be given via definition, while the domains of non-primitive types are originated from them. Non-primitive types are usually called functor types.

Definition 1. *Let PT be an arbitrary set of symbols, the set of primitive types, such that $o \in PT$. Then the set $TYPE_{PT}$ is defined inductively as follows:*

1. $PT \subseteq TYPE_{PT}$;
2. $\alpha, \beta \in TYPE_{PT} \Rightarrow \langle\alpha, \beta\rangle \in TYPE_{PT}$.

Remark 1. Here o is the type of formulae from the syntactic point of view, and the type of their possible semantic values from the semantic point of view. $\langle\alpha, \beta\rangle$ is the type of functors which, when they are filled in by an argument of type α, yield an expression of type β in syntax (in the formal language), and it stands for the type of function from objects of type α to objects of type β in semantics.

The type–theoretical language is the most general one concerning the functor–argument decomposition. It has only two syntactic operations: filling a functor with an argument (function application from the semantic point of view) and lambda abstraction. The latter produces a way how to create a functor from an expression.

Definition 2. *A* type–theoretical language *is an ordered quadruple*

$$L = \langle LC, Var, Con, Cat\rangle$$

satisfying the following conditions:

1. *LC is the set of theoretical constants.*[5] $LC = \{\lambda, (,)\}$
2. $Var = \cup_{\alpha \in TYPE_{PT}} Var(\alpha)$ *and* $Var(\alpha)$ *is a denumerably infinite set of symbols*[6].
3. $Con = \cup_{\alpha \in TYPE_{PT}} Con(\alpha)$, *where* $Con(\alpha)$ *is a denumerably set of symbols.*[7]
4. *All mentioned sets of symbols are assumed to be pairwise disjoint ones.*

[5] A theoretical constant has the same semantic value (or sense) in every interpretation as a logical constant does in a logical system.

[6] $Var(\alpha)$ is the set of variables of the type α.

[7] Con is the set of non–theoretical symbols of L. The semantic value of an expression belonging to the set Con is given by an interpretation.

5. $Cat = \cup_{\alpha \in TYPE_{PT}} Cat(\alpha)$, *where the sets* $Cat(\alpha)$ *are defined by the inductive rules (a)...(c) as follows*[8]*:*
   (a) $Var(\alpha) \cup Con(\alpha) \subseteq Cat(\alpha)$;
   (b) $C \in Cat(\langle\alpha,\beta\rangle)$, $B \in Cat(\alpha) \Rightarrow$ '$C(B)$' $\in Cat(\beta)$;
   (c) $A \in Cat(\beta)$, $\tau \in Var(\alpha) \Rightarrow$ '$(\lambda\tau A)$' $\in Cat(\langle\alpha,\beta\rangle)$;

The (total or partial) functor–argument frame is the compositional mirror of type–theoretical language. It can be said that the functor–argument frame gives *possible* semantic values.

Definition 3. *A* total functor–argument frame F *is the system of sets* $\langle Dom_F(\gamma)\rangle_{\gamma \in TYPE_{PT}}$ *such that*

1. *If* $\gamma \in PT$, *then* $Dom_F(\gamma)$ *is an arbitrary nonempty set.*
2. $Dom_F(\langle\alpha,\beta\rangle) = Dom_F(\beta)^{Dom_F(\alpha)}$ *for all* $\langle\alpha,\beta\rangle \in TYPE_{PT}$

Definition 4. *A* partial functor–argument frame PF *is the system of sets* $\langle Dom_{PF}(\gamma)\rangle_{\gamma \in TYPE_{PT}}$ *such that*

1. *if* $\gamma \in PT$, *then* $Dom_{PF}(\gamma)$ *is an arbitrary set with a distinguished member* Θ_γ, *which is called the null entity of type* γ, *such that* $Dom_{PF}(\gamma) \setminus \{\Theta_\gamma\} \neq \emptyset$;
2. $Dom_{PF}(\langle\alpha,\beta\rangle) = Dom_{PF}(\beta)^{Dom_{PF}(\alpha)}$ *for all* $\langle\alpha,\beta\rangle \in TYPE_{PT}$ *and* $\Theta_{\langle\alpha,\beta\rangle} = g$ *where* $g \in Dom_{PF}(\langle\alpha,\beta\rangle)$ *and* $g(u) = \Theta_\beta$ *for all* $u \in Dom_{PF}(\alpha)$.

Interpretive function and assignment associate the constants and the variables of the type–theoretical language with their semantic values. In a model, which consists of a frame, an interpretive function and an assignment, semantic rules can be defined to determine the semantic values of compound expressions with respect to the given model.

Definition 5. *A* (total or partial) model M *on* G *is an ordered triple* $\langle G, \varrho, v\rangle$ *where*

1. G *is a (total or partial) functor–argument frame;*
2. ϱ, v *are functions with domains* Con *and* Var *respectively*[9] *such that*
   (a) *if* $a \in Con(\alpha)$, *then* $\varrho(a) \in Dom_G(\alpha)$;
   (b) *if* $\tau \in Var(\alpha)$, *then* $v(\tau) \in Dom_G(\alpha)$.

Remark 2. A model M on G is total or partial if G is a total or partial functor–argument frame respectively.

If $M = \langle F, \varrho, v\rangle$ is a total model on F, then

$$Dom_M(\alpha) = Dom_F(\alpha).$$

If $PM = \langle PF, \varrho, v\rangle$ is a partial model on PF, then

$$Dom_{PM}(\alpha) = Dom_{PF}(\alpha) \setminus \{\Theta_\alpha\}.$$

If M $(= \langle G, \varrho, v\rangle)$ is a total or partial model, $\xi \in Var(\gamma)$ and $u \in Dom_G(\gamma)$, then the model M^u_ξ $(= \langle G, \varrho, v[\xi : u]\rangle)$ is like M except that $v[\xi : u](\xi) = u$.

[8] Cat is the set of all well–formed expressions of L. The set $Cat(\alpha)$ is the α–category of L ($\alpha \in TYPE_{PT}$).

[9] ϱ is an interpretive function, v is an assignment.

Definition 6. *A total or partial model* M $(= \langle G, \varrho, v\rangle)$ *assigns each expression* A *of type* α *a* semantic value $[\![A]\!]_M$ *according to following semantic rules:*

1. *if* $a \in Con(\gamma)$*, then* $[\![a]\!]_M = \varrho(a)$*;*
2. *if* $\xi \in Var(\gamma)$*, then* $[\![\xi]\!]_M = v(\xi)$*;*
3. *if* $A \in Cat(\langle\alpha, \beta\rangle)$ *and* $B \in Cat(\alpha)$*, then* $[\![A(B)]\!]_M = [\![A]\!]_M([\![B]\!]_M)$*;*
4. *if* A *is an expression of type* β *and* $\xi \in Var(\alpha)$*, then* $[\![\lambda\xi A]\!]_M = g$*, where* g *is a function from* $Dom_G(\alpha)$ *to* $Dom_G(\beta)$ *such that* $g(u) = [\![A]\!]_{M^u_\tau}$ *for all* $u \in Dom_G(\alpha)$*.*

Proposition 1. *If* M *is a total model and* $A \in Cat(\alpha)$*, then* $[\![A]\!]_M \in Dom_M(\alpha)$*. If* M *is a partial model, then* $[\![A]\!]_M \in Dom_M(\alpha) \cup \{\Theta_\alpha\}$*.*

Definition 7. *If* M *is a total or partial model, then* A *is* meaningful *with respect to* M*, in symbols* $A \in Cat^M_{mf}$ *if* $A \in Cat(\alpha)$ *for some type* α *and* $[\![A]\!]_M \in Dom_M(\alpha)$*.*

Remark 3. If M is a total model, then all $A \in Cat$ are meaningful, i.e. there is no difference at all between the notions of well–formedness and meaningfulness. We can only make a real differentiation between them in the case of partial models.

Theorem 1. *If* $A \in Cat$*,* $M_1 = \langle G, \varrho, v_1\rangle$ *and* $M_2 = \langle G, \varrho, v_2\rangle$ *are two (total or partial) models of* L *with the same frame* G *and interpretive function* ϱ *such that* $v_1(\tau) = v_2(\tau)$ *for all* $\tau \in V(A)$[10]*, then* $[\![A]\!]_{M_1} = [\![A]\!]_{M_2}$*.*

Proposition 2. *If* $A \in Cat$ *is a closed expression, then* $[\![A]\!]_M$ *is independent from* v *i.e* $[\![A]\!]_M = [\![A]\!]_{M^u_\tau}$ *for all* $\tau \in Var(\gamma)$ *and* $u \in Dom_F(\gamma)$*.*[11]

To prove lambda–conversion law we need Law of replacement 2 and Lemma 1. The first one says that in semantics we only take into consideration semantic values and don't pay any attention to the expression itself — except its type — whose semantic value is given. It doesn't matter how we get a semantic value, what form of the compound expression gets the semantic value. We may formulate the property in the law of replacement by means of universal replacement of expressions belonging to the same type with the same semantic value. From the logical–philosophical point of view, the law of replacement is a special type–theoretical formulation of a version of the principle of compositionality called the substitutivity principle, which goes back to Leibniz.

> The Substitutivity Principle: "If two expressions have the same meaning, then substitution of one for the other in a third expression does not change the meaning of the third expression." [Szabo, 2000, p. 490]

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[10] The definitions of subterms, free variables, open and close expressions and the substitutability are usual ones. The set $V(A)$, is the set of free variables of the expression A.

[11] In the case of closed expressions we can speak about models as ordered pairs of frames and interpretive functions.

I must emphasize that the law of replacement can only be considered as a restricted version of the substitutivity principle, the unrestricted form of the substitutivity principle holds only in Husserlian models.The next definition introduces the notion of 1–compositionality. 1–compositional systems fulfill a restricted version of the substitutivity principle, and Corollary 1 of Law of replacement 2 says that our general system is compositional in the sense of 1–compositionality.

Definition 8. *Let M be a model of L. We say that M is* 1–compositional *if for all well–formed expressions A, B, C $(A, B, C \in Cat)$ and variable τ $(\tau \in Var)$ such that $(\lambda\tau C)(A), (\lambda\tau C)(B) \in Cat^M_{mf}$*

$$[\![A]\!]_M = [\![B]\!]_M \Rightarrow [\![(\lambda\tau C)(A)]\!]_M = [\![(\lambda\tau C)(B)]\!]_M$$

Theorem 2 (Law of replacement). [12]

If $A \in Cat$, $B, C \in Cat(\gamma)$, and M is a (total or partial) model of L, then

$$[\![B]\!]_M = [\![C]\!]_M \Rightarrow [\![A]\!]_M = [\![A[C \downarrow B]]\!]_M.$$

Corollary 1. *If M is a (total or partial) model of L, then M is 1–compositional.*

Lemma 1. *If B is substitutable for variable τ in A, M is a (total or partial) model, and $[\![B]\!]_M = u$, then $[\![A^B_\tau]\!]_M = [\![A]\!]_{M^u_\tau}$.*

Theorem 3 (Lambda–conversion law).

If $A \in Cat$, $\tau \in Var(\beta)$, $B \in Cat(\beta)$ and B is substitutable for τ in A, then $[\![(\lambda\tau A)(B)]\!]_M = [\![A^B_\tau]\!]_M$ for all (total or partial) models M.

It is needless to say that a type theoretical language with its possible models does not constitute a logical system, since the notion of functor–argument frame is too universal, logically relevant semantic values cannot appear in it. Therefore, there is no real opportunity to give the notion of logically valid inferences, consequence relations. However, at the same time there are many different possible ways to modify functor–argument frames in order to get logical systems. In the construction of logical systems I will show one of the most general such ways which is especially relevant from the logical–philosophical point of view.

4 The most elementary cases

A type–theoretical language with identity is closer to constituting a logical system than a type-theoretical language without identity since in the former some semantic values (of identity sentences) appear which look like logically relevant semantic values. From a theoretical point of view it is not problematic to define a very simple logical system for *identity sentences*. The received system would be very similar to classical propositional logic (in the total case), and to propositional logic allowing truth value gaps (in the partial case). These cases are so simple that we will not deal with them.

[12] If $A \in Cat$ and $B, C \in Cat(\gamma)$, then $A[C \downarrow B]$ $(\in Cat)$ is obtained by replacing a subterm occurrence (i.e. not preceded immediately by λ) of B by C.

The proper question is what can be said about the semantic values of sentences in the light of the possible semantic values of identity sentences. If we try to follow a very simple method, we can embed identity sentences into the set of sentences (i.e. we can suppose that $Cat(o_=) \subset Cat(o)$). If we focus on the total case, then in the semantic definition of total frames we may introduce a stronger condition: $D(o) = D(o_=)$, (i.e. for example in this case there are only two possible semantic values for a sentence: 0 or 1, in other words it may be true or false). On the one hand this means that in syntax there is no need to differentiate sentences from identity sentences, we can avoid introducing $o_=$, the type of identity sentences, and on the other hand, that in semantics the senses (meanings) of sentences can appear only in a very restricted manner: formulae may have 0 or 1 as semantic values. It is obvious that the received systems will be different versions of the logical system which is usually called extensional (type–theoretical) logic. The decision concerning the possible semantic values of sentences outlined above dominates the whole system and it has serious consequences. More specifically 'real' senses of sentences disappear and only one aspect of sense can be handled: whether a sentence with a given sense is true or false in a fixed context. The received system can be used to represent well-known extensional properties.

5 Context as a bridge between different sorts of semantic values

The next step is to introduce the notion of context. This step is very important, since it provide a real possibility to represent sentential sense which differs significantly from the 'extensional sense'. (At first we will only deal with sentences.) As it was emphasized many times in previous chapters senses are the primary semantic values. They have many different roles, but one of these is especially important: If a sentence has any other semantic value besides sense, then its sense has to determine the other semantic value. Semantic values of identity sentences are appropriate candidates for other semantic values of sentences. Therefore, if a sentence in general may have the same semantic value as an identity sentence, then the sense of the sentence has to determine it.

How can the sense of a sentence determine its other semantic value? The answer for this question can be found in those situations when we need these values or when we use them, and so it is very straightforward: only in the case of uttering the given sentence are we interested this other value of the sentence. A sentence utterance can only be grasped in connection with utterances of other sentences. From a theoretical point of view, usually a set of utterances of given sentences is considered to be a simple representation of context.

How can we represent a context? The natural (and usual) way is to provide those sentences which are true in the context in question (or to specify which sentences are true, which are false and which are irrelevant). Obviously, this depends on the senses of the given sentences, hence the precise formulation is the following: a context is a special representation which is based on a set of senses of the relevant sentences, and which also includes specifying the truth values of those sentences.

In a formal model, the main component of a context is a function from the set of senses of sentences to the set of semantic values of identity sentences (i.e. to the set of possible truth values). Up to this point we have dealt with sentences only, but, as expected, there is no theoretical difference with respect to the behaviour of expressions of other primitive types. Expressions of primitive types other than the type of sentences play a similar and crucial role in constructing the notion of context as sentences. (Almost the same can be said about these expression as about sentences, however, there is no such aid that could be compared to the one provided by identity sentences.) In order to define the general notion of context we have to specify the sets of 'secondary' semantic values of expressions in the case of every primitive type. We will call 'secondary' semantic values extensions (or factual values using Ruzsa's original terminology). The set of extensions of a given primitive type γ will be denoted by $D_{ext}(\gamma)$.

Definition 9. *A* system of extensions of primitive type(s) *is the system of sets*

$$\langle D_{ext}(\gamma)\rangle_{\gamma\in PT}$$

such that

(a) $D_{ext}(o) = \{0, 1, 2\}$, $\Theta_o^{ext} = 2$ *(*Θ_o^{ext} *the extensional null entity of type* o*);*
(b) if $\gamma \in PT$ *and* $\gamma \neq o$*, then* $D_{ext}(\gamma)$ *is an arbitrary set with a distinguished member* Θ_γ^{ext}*, which is called the extensional null entity of type* γ*;*

Definition 10. *Let* G $(= \langle Dom(\gamma)\rangle_{\gamma\in TYPE_{PT}})$ *be a (total or partial) frame, and* SE $(= \langle D_{ext}(\gamma)\rangle_{\gamma\in PT})$ *be a system of extensions of primitive type(s). A* contextual function for the frame G *relying on the system of extensions* SE *is a function* C_G *such that*

(a) the domain of the function C_G *is* $\cup_{\gamma\in PT} Dom(\gamma)$*;*
(b) if $u \in Dom(\gamma)$*, then* $C_G(u) \in D_{ext}(\gamma)$ $(\gamma \in PT)$*.*

In logically relevant cases not only a frame is needed, but also a context for the frame. The the next definition introduces the notion of a context for a frame.

Definition 11. *A (total or partial)* context for a frame G *is an ordered triple*

$$\langle G, SE, C_G\rangle$$

where

(a) G $(= \langle Dom(\gamma)\rangle_{\gamma\in TYPE_{PT}})$ *is a (total or partial) frame;*
(b) SE $(= \langle D_{ext}(\gamma)\rangle_{\gamma\in PT})$ *is a system of extensions of primitive type(s);*
(c) C_G *is a contextual function for the frame* G *relying on the system of extensions* SE*.*

In Definition 5 we introduced a general notion of models. By means of the notion of a context for a frame the notion of logically relevant models can be introduced.

Definition 12. *A* logically relevant (total or partial) model $[M_C]$ *is an ordered triple*

$$\langle CFF, \varrho, v\rangle$$

where

(a) CFF $(= \langle G, SE, C_G\rangle)$ *is a (total or partial) context for the frame* G*;*
(b) ϱ, v *are functions as in Definition 5.*

If we define central logical notions as satisfiability, unsatisfiability, consequence relation and validity by means of context sensitive frames and logically relevant (total or partial) models, we get very strong notions.

Definition 13. *Let* Γ *be a set of formulae, i.e.* $\Gamma \subset Cat(o)$ *and* A *a formula, i.e.* $A \in Cat(o)$*.*

(a) Γ *is satisfiable if there is a logically relevant model* M_C *such that* $C_G([\![A]\!]_{M_C}) = 1$ *for all* $A \in \Gamma$*,*
where $M_C = \langle CFF, \varrho, v\rangle$ *and* $CFF = \langle G, SE, C_G\rangle$
(b) The set Γ *is unsatisfiable if it is not satisfiable.*
(c) A *is a logical consequence of* Γ $(\Gamma \vDash A)$ *if the set,* $\Gamma \cup \{\neg A\}$ *is unsatisfiable.*
(d) A *is valid* $(\vDash A)$ *if* $\emptyset \vDash A$*.*
(e) A *is irrefutable if there is no logically relevant model* M_C *such that* $C_G([\![A]\!]_{M_C}) = 0$*.*

Logically relevant intensional models provide a new level where different features of sense can be represented. The main idea is that the possible context can be determined in logically relevant intensional models.

Definition 14. *Let* $\langle G, \varrho, v\rangle$ *be a (total or partial) model, and* SE *be a system of extensions,* C^i_G *be a contextual function from* G *to* SE *for* $i \in I$*, where* I *is an arbitrary nonempty set. A* logically relevant intensional (total or partial) model $[M^{int}_C]$ *is the set of ordered triples*

$$\{\langle CFF_i, \varrho, v\rangle : i \in I\}$$

where $CFF_i = \langle G, SE, C^i_G\rangle$ *is a (total or partial) context for the frame* G*.*

By means of logically relevant intensional models a great number of 'classical' intensional features can be represented. For example, extensionality can be represented on two different levels: as extensionality in a context, and as extensionality in a logically relevant intensional model.

6 An example: 'classical' intensional logic

In this section we will show how to reconsider 'classical' intensional logic in the light of our general investigation. At the same time one can recognize the theoretical sources of our notion of context, and one can imagine its various theoretical role.

Following the traditional method, we may suppose that only two symbols belong to the set of primitive types, type o, i.e. the type of formulae as it appears in Definition 1, and type ι, the type of individual names. The system of types generated by o and ι as primitive types will be denoted by $TYPE_{Fr}$. In what follows we can suppose that our language is a type–theoretical language based on $TYPE_{Fr}$.

The next question is how to define frames relying on the standard method, which proceeds from extensions to intensions. From a general point of view, sense is the primary semantic value, hence we have to define the frame of logically relevant senses, i.e. the frame of intensions. Following the method of possible world semantics we can say that the intension of a formula is the rule that determines whether the formula expresses a true or a false statement in a given situation (world). This rule can represent the truth conditions of a formula. The intension of an individual name is the rule which determines its reference in a given situation (world).

An intensional functor–argument frame is a functor–argument frame such that

- The set of primitive types contains type ι, the type of individual names, and type o, the type of formulae.
- The rules mentioned above, which serve as intensions, are functions from the set of indices to the set of objects or truth values in the case of primitive types, and from the semantic domain of the input to the semantic domain of the output otherwise.

Definition 15. *By an* intensional functor–argument frame F_{int} *let us mean an ordered triple*

$$F_{int} = \langle U, I, D_{int} \rangle$$

satisfying the following conditions:

1. $D_{int}(o) = \{0, 1\}^I$;
2. $D_{int}(\iota) = U^I$;
3. $D_{int}(\langle \alpha, \beta \rangle) = D_{int}(\beta)^{D_{int}(\alpha)}$ *for all* $\langle \alpha, \beta \rangle \in TYPE_{Fr}$

$M \vDash_i A$ means that the function, which is the semantic value of formula A ($A \in Cat(o)$) with respect to M, is 1 at i, i.e. $[\![A]\!]_M(i) = 1$. Using intensional functor–argument frames we can introduce one of the simplest notions of logical consequence. Obviously it has to be presupposed that 0 and 1 have special logical roles or logical "meanings". 1 indicates that a sentence has the property preserved by the intended notion of consequence relation. For the sake of simplicity we can say that 1 and 0 correspond to truth and falsity, respectively. I have to emphasize that there is no need to say anything about the nature of truth values here.

Definition 16. *$\langle F, \varrho, v, i\rangle$ is said to be a* true intensional representation *of Γ ($\subseteq Cat(o)$) if*

1. *F ($= \langle U, I, D_{int}\rangle$) is an intensional functor–argument frame;*
2. *$\langle F, \varrho, v\rangle$ ($= M$) is a model on F;*
3. *$i \in I$;*
4. *$M \vDash_i A$ for all $A \in \Gamma$.*

Definition 17. *Suppose that $\Gamma \subseteq Cat(o)$ and $A \in Cat(o)$. A is a strong semantic consequence of Γ ($\Gamma \Vdash A$) if A is true, i.e. $M \vDash_i A$ in every true intensional representation of Γ.*

In the framework outlined above the semantic value of any formula is a sentence intension, and we can speak about truth and falsity, since sentence intensions are functions from indices to truth values. Therefore, sentences (and individual names) have two different sorts of semantic value. In the first place they have intensions (corresponding to their informal senses) and in the seocnd place formulae have truth values (and individual names have reference) at a given index. However, only intensions of compound type expressions are present. A natural question arises here: is there any connection between the truth values of two formulae if one of them involves the other as a subformula? From a general point of view the answer is 'no' or at least 'it depends'. However, in special cases we may recognize some deterministic connection between the semantic values in question. In order to get the whole picture we will use the well–known family of extensional semantic values.

Definition 18. *By an* extensional functor–argument frame *F_{ext} let us mean an ordered pair*

$$F_{ext} = \langle U, D_{ext}\rangle$$

satisfying the following conditions:

1. *U is an arbitrary non–empty set;*
2. *$D_{ext}(\iota) = U$;*
3. *$D_{ext}(o) = \{0, 1\}$;*
4. *$D_{ext}(\langle\alpha, \beta\rangle) = D_{ext}(\beta)^{D_{ext}(\alpha)}$ for all $\langle\alpha, \beta\rangle \in TYPE_{Fr}$*

Remark 4. The difference between intensional and extensional functor–argument frames is manifested only in the definitions of domains of primitive types. In extensional cases, where M is a model on an extensional functor–argument frame, if $A \in Cat(o)$, then $M \vDash A$ means that $[\![A]\!]_M = 1$.

Definition 19. *A model $M = \langle F, \varrho, v\rangle$ on F is said to be a* true extensional representation *of Γ ($\subseteq Cat(o)$) if*

1. *F is an extensional functor–argument frame;*

2. $M \models A$ *for all* $A \in \Gamma$*.*

Definition 20. *Suppose that* $\Gamma \subseteq Cat(o)$ *and* $A \in Cat(o)$*.* A *is a strong semantic consequence of* Γ ($\Gamma \Vdash A$) *if* A *is true with respect to* M *i.e.* $M \models A$ *in every true extensional representation* M *of* Γ*.*

We have a type–theoretical language, and two different notions of frames, intensional and extensional. Both contain logically relevant semantic values at least for sentences and individual names. The semantic values of compound type expressions are generated from the semantic values of primitive type expression by the principle of contextuality.

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