Uniwersytet Ekonomiczny

George Matysiak

Introduction to Portfolio Theory
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Agenda

- MPT, risk and return
- Diversification
  - Naïve risk reduction
  - Efficient Frontier

Risk and return

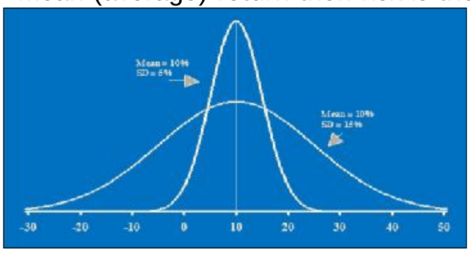
- A key investment indicator is expected total return
- A second important investment indicator is risk
- Risk is a measure of the probability of expected return not being achieved
- Traditional measure of risk is *variance* or *standard deviation* of expected returns

What is Risk?

- The possibility that *actual* return will differ from *expected* return
- Uncertainty in the distribution of possible outcomes

A Measure of Risk?

If expected return is measured as by the mean (average) return then risk is the



Variance, Standard Deviation and Correlation

Statistics for Portfolio Inputs

- Expected Return
- Variance/Standard Deviation
- Covariance/Correlation

Variance and Standard Deviation

The standard deviation of a data set measures the spread or dispersion of the data. The square of this is known as the *variance*. Given a sample of data, x_i, in order to obtain an unbiased estimate of the underlying population variance, s_x^2, from which the sample was drawn, the following formula is used:

$$s_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

where $\bar{x}$ is the mean (average) of the data.

The square root of this is known as the *standard deviation*.

Covariance

The covariance is a measure of association between two variables. If there are two random variables X and Y with mean (expected) values of $\bar{x}$ and $\bar{y}$ respectively, the covariance is defined as:

$$\text{cov}(x, y) = E([(X - \bar{x})(y - \bar{y})]$$

The covariance is a measure of association between two variables. Given a sample of observed data the covariance is calculated from:

$$\text{cov}(x, y) = \frac{\sum_{i=1}^{n}[(X_i - \bar{x})(Y_i - \bar{y})]}{n-1}$$

Note: In Excel multiply Covar by (n/n-l) as only population Covar is calculated.

Correlation

The value of the covariance depends on the units in which X and Y are measured i.e. it is scale dependent. To correct for this it is divided by the standard deviations of the two variables and this ratio becomes a pure number. The resulting ratio is known as the *correlation coefficient*, r_{xy} .

So, the correlation coefficient, r_{xy}, is found from:

$$r_{xy}\ Z\ \frac{\text{cov}(x,y)}{\pm_x \pm_y}$$

where, $\pm_x$ and $\pm_y$ are the sample standard deviations of x and y respectively.

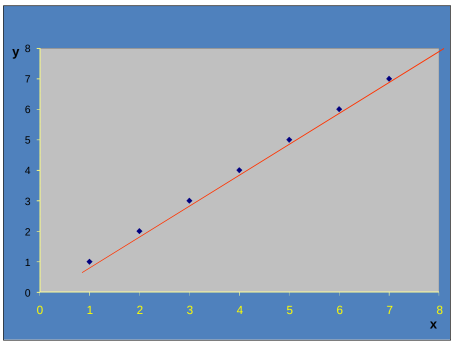
It can be shown that the correlation coefficient lies in the interval -1 to +1 i.e.

$$J1 \text{ á } r_{xy} \text{ á } H1$$

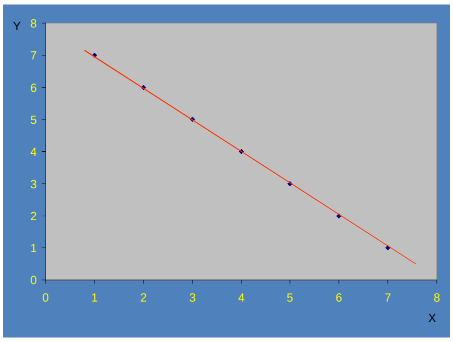
Correlation

- The strength of a relationship between two variables is measured by the coefficient of correlation, r
- Perfect positive correlation occurs when r = 1
- Perfect negative correlation occurs when r = -1
- No correlation occurs when r = 0

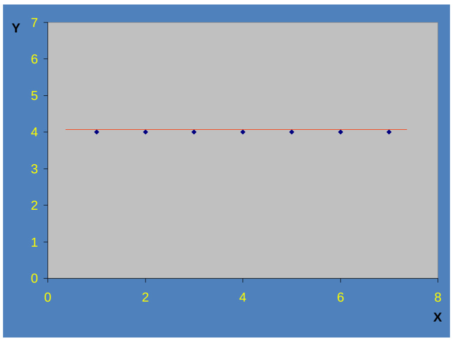
Perfect Positive Correlation



Perfect Negative Correlation



No correlation



MPT

- Over the last 20 years or so practitioners in property markets have begun to look at portfolios in a more structured/formal manner
- Whilst it is generally acknowledged that diversification is desirable, purchases and sales were (are) undertaken on a deal-by-deal basis without formally tracing through the wider implication/impact on *diversification* within the portfolio
- The essential concept of portfolio theory is that the risk of any individual asset *as a component* of a portfolio is different from its risk in isolation

MPT

Portfolio theory concerns itself with focussing attention on the risks of a portfolio, which is regarded as an entity

MPT

- In general, the risk of a portfolio will be less than the average of the individual components. This stems from individual assets being *less than perfectly correlated*
- MPT techniques allow portfolios to be managed in a way that the risk of a portfolio can be controlled. It deals with the selection of *optimal portfolios* by rational risk-averse investors
- The objective in constructing portfolios is to determine the appropriate combination (weights) of assets which yields a desired risk and return trade-off

Assumptions about investors' preferences

- We will assume that investors
  - *prefer mean/expected return*
  - *prefer lower standard deviation in return*
  - *do not care about other things*
- This means that standard deviation/variance is the relevant measure of risk

Basic idea…

- Selecting assets which have low correlation is a central aspect of Modern Portfolio Theory
- The objective of MPT is either
  - ➢for a given level of risk, to achieve the maximum return

  or
  - ➢for a given return, to achieve the minimum risk
- The output of the analysis is the proportion of funds to be invested in each asset, and a measure of the expected return and the risk

Mean-Variance Framework Basic Measures

- Requirement:
  - expected return
  - variance
  - covariance/correlation

How do we obtain these numbers ?

Diversification

- Risk reduction is a well understood concept
  - Don't put all your eggs into one basket: diversify risk!

Diversification

One of the few areas in economics where you get a 'free lunch' i.e. less risk without necessarily reducing expected return!

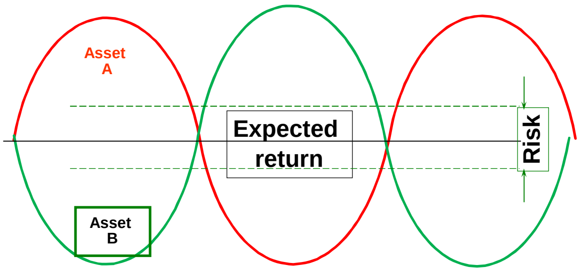
The mean/variance hypothesis

- More return is better than less return
- Less risk is better than more risk
- Investment A is better than investment B if, and only if, its expected return is higher and its risk is equal to or less than that of investment B
- However, a more rational approach may be to combine, *holding both A and B*

Portfolios

- Combining several securities into a portfolio can reduce overall risk
- How does this work?

Risk and Return Profile
Multi Asset (Portfolio)



Assets A & B are negatively correlated

Diversification

- Investing in more than one security to reduce risk
- If two stocks are perfectly positively correlated, diversification has no effect on risk
- If two stocks are perfectly negatively correlated, the portfolio is perfectly diversified

Portfolios of Securities: Return

- Investors' opportunity set is comprised not only of sets of individual securities but also combinations, or *portfolios*, of securities
- The achieved return on a portfolio is the weighted average of returns on component portfolios/securities:

$$R_{pt} = \sum_{i=1}^{N} w_i R_{it}$$

- The expected return is also a weighted average

$$E(R_{pt}) = \sum_{i=1}^{N} w_i E(R_{it})$$

Portfolios of Securities: Risk

- *However*, the standard deviation of a portfolio is NOT simply a weighted average of securities standard deviations
- We also need to account for co-variances
- Example with 2 risky securities X and Y

- Will the portfolio s.d. be higher or lower than a simple weighted average?

$$\pm_p^2 = w_x^2 \pm_x^2 + w_y^2 \pm_y^2 + 2 w_x w_y \mathrm{Cov}(xy)$$

$$\pm_p^2 = w_x^2 \pm_x^2 + w_y^2 \pm_y^2 + 2 w_x w_y \pm_x \pm_y \, ^\circ_{xy}$$

Example

Risk Diversification

	Share of Investment (weight)	Risk (SD)
Asset A:	0.5	8
Asset B:	0.5	8
Correlation:	0.5	

Portfolio consisting of both assets

Total Volatility:	48	
Standard Dev:	6.928203 < 8	

Example (continued)

Risk Diversification

Different correlation values

Correlation	Portfolio Risk (SD %)
-1	0
-0.75	2.828427125
-0.5	4
-0.25	4.898979486
0	5.656854249
0.25	6.32455532
0.5	6.92820323
0.75	7.483314774
1	8

Observation

Combinations of less than perfectly correlated assets result in risk reduction

Portfolio Risks and Returns

Assume we are considering investment in the following:

Investment	Expected Return %	Risk SD %
A	10	20
B	15	30

- If we want good returns we should invest in?
- If we want low risk we should invest in?
- But if we want high returns and low risk we should do what?

Portfolio Expected Returns

$$E(R_{Port}) = \sum_{i=1}^{m} w_i E(R_i)$$

where

w_i = weight associated with asset i

$E(R_i)$ = Expected Return of asset i

m = number of asset classes

E(Rp) = (0.4 x 10) + (0.6 x 15) = 13%

Portfolio Risk: The Two-Asset Case

$$\pm_p \ Z \ \sqrt{\pm_1^2 w^2 \ H \ \pm_2^2 w^2 \ H \ 2wwCov_{1,2}}$$

Remember $r_{1,2}$=$Cov_{1,2}/s_1*s_2$

So $Cov_{1,2}$=$r_{1,2}*s_1*s_2$

$$\pm_p \ Z \ \sqrt{\pm_1^2 w^2 \ H \ \pm_2^2 w^2 \ H \ 2ww°_{1,2}\pm_1\pm_2}$$

Perfect Positive Correlation r =+1

$$Sp^2 = w_1^2 S_1^2 + w_2^2 S_2^2 + 2w_1 w_2 (+1) S_1 S_2$$
$$Sp^2 = (w_1 S_1 + w_2 S_2)^2$$
$$Sp = w_1 S_1 + w_2 S_2$$

From the example:

Sp = (0.4 x 20) + (0.6 x 30)
= 26.0%

Zero Correlation r = 0

$$Sp = \sqrt{w_1^2 S_1^2 + w_2^2 S_2^2}$$

From the example:

$$Sp = \sqrt{(0.4^2 x 20^2) + (0.6^2 x 30^2)}$$
$$Sp = \sqrt{388}$$
$$Sp = 19.7\%$$

Think about investing in different sectors/regions/other groupings?

Different sectors/different regions low correlation?

Perfect Negative Correlation r = -1

$$Sp^2 = w_1^2S_1^2 + w_2^2S_2^2 + 2w_1w_2(-1)S_1S_2$$
$$Sp^2 = (w_1S_1 - w_2S_2)^2$$
$$Sp = \sqrt{(w_1S_1 - w_2S_2)^2}$$

From the example:

$$Sp = \sqrt{(0.4x20 - 0.6x30)^2}$$
$$Sp = 10.0\%$$

Anorak's corner: portfolio risk reduction

- For N securities, in general, the formula is:

$$\pm_p{}^2 = \sum_{i=1}^{N} w_i{}^2 \pm_i{}^2 + \sum_{i=1}^{N}\sum_{\substack{j=1 \\ i \neq j}}^{N} w_i w_j \pm_{ij}$$

- The first term is a square weighted average of securities variances; the second term captures N(N-1) covariance terms, $Cov(i, j) = \pm_{ij}$
- Intuitively, what happens to the portfolio's variance as N gets large?

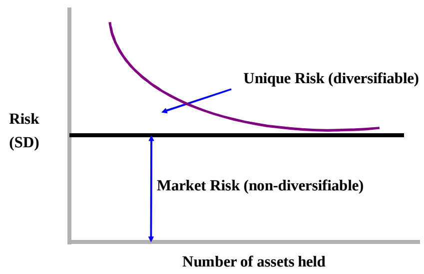
Anorak's corner: portfolio risk reduction

- The variance of an equally-weighted portfolio is given by:

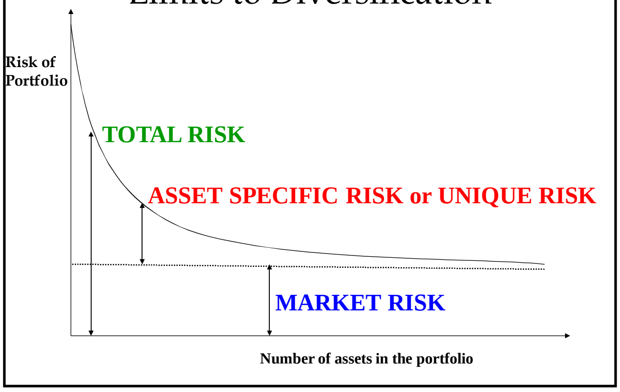
$$\pm_P^2 = \frac{1}{N}\begin{bmatrix} Average \\ Variance \end{bmatrix} + \left\{1 - \frac{1}{N}\right\}\begin{bmatrix} Average \\ Co\ var\ iance \end{bmatrix}$$

- When the number of securities in the portfolio increases, what matters is the covariance term!
- *In the case of a well-diversified portfolio, the variance will be equal to the average covariance between the individual components*

Risk reduction by Naïve diversification



Limits to Diversification



Portfolio Risk

- The total risk of a portfolio (indeed of a security) consists of two parts:
  - **Market (or systematic) Risk**
  - **Unique (or firm-specific) Risk**
- **Diversification reduces the Unique Risk…**

↓

- **…hence, diversification reduces total risk**

Some risk can be diversified away and some cannot

- *Market risk* (*systematic risk)* is *nondiversifiable.* This type of risk cannot be diversified away
- Asset-unique or property unique risk *(unsystematic risk)* is *diversifiable*. This type of risk can be reduced through diversification

Partitioning risk

- Total risk (volatility) can be decomposed into components
  - returns accounted for by common factors
  - idiosyncratic component

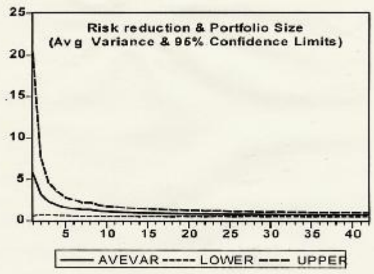
Risk reduction opportunities: intra-sector

Sector	Average correlation between properties	
	1988-1992	1993-1997
Retail	0.134	0.144
Office	0.213	0.128
Industrial	0.228	0.098

Risk reduction opportunities: inter-sector

Sector	Average correlation between properties	
	1988-1992	1993-1997
Retail - Office	**0.129**	**0.130**
Retail - Industrial	**0.132**	**0.113**
Office - Industrial	**0.213**	**0.115**

Risk reduction in property portfolios



Percentage reduction in risk for each sector

Percentage reduction in risk assuming equal levels of investment in each sector

No. of Properties	% of reduction in risk			
	Retail	*Office*	*Industrial*	*Portfolio*
1	0	0	0	0
2	26	26	27	26
3	36	37	38	37
4	42	43	44	43
5	46	47	49	48
10	56	57	59	57
20	61	63	65	63
30	63	65	67	65
40	64	66	68	66
50	64	66	69	67
100	66	68	70	68
1000	67	69	72	69

Property portfolio diversification

No of Props	Office	Retail	Industrial
1	0.11	0.08	0.07
2	0.20	0.14	0.13
3	0.28	0.20	0.18
4	0.34	0.25	0.23
5	0.39	0.29	0.27
10	0.56	0.45	0.42
20	0.72	0.62	0.59
30	0.79	0.71	0.69
40	0.84	0.77	0.75
50	0.86	0.80	0.79
100	0.93	0.89	0.88
200	0.96	0.94	0.94
1000	0.99	0.99	0.99

Fund managers rely on either real estate *type* or *location* as the dominant criterion for portfolio construction

Diversification

- Sector and regional diversification the best overall
- But sector diversification closest to overall 'efficient frontier' (provides the best risk-return trade-off)
- Identification of more refined sectors and regions will offer the greatest diversification benefits!

Portfolio risk and return

- Selecting assets which have low correlations is a central aspect of Modern Portfolio Theory
- The objective of MPT is either
  – for a given level of risk, to achieve the maximum return

  or

  – for a given return, to achieve the minimum risk
- The output of the analysis are the proportions to be invested in each asset, providing a measure of portfolio expected return and the risk

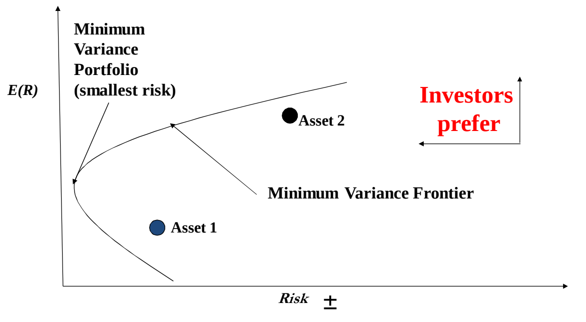
Efficient Portfolios

- Understanding the return and risk attributes of individual securities and the correlations between securities allows us to construct *efficient* combinations which attempt to "reduce risk as much as possible for a given level of expected return." How do we do it?
- We work in a mean-variance framework
  – assumes all investors prefer higher returns, all else equal
  – assumes all investors prefer lower risk, all else equal

Delineating Efficient Portfolios

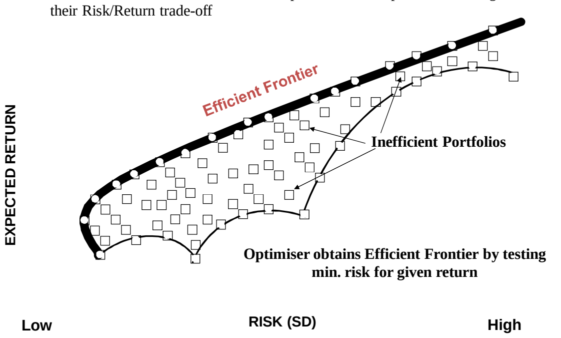
- Rational approach to construction of portfolios
- An efficient portfolio *maximises return* for a given level of risk or *minimises risk* for a given level of return
- Investors will seek optimal risk/return combinations ≡ efficient portfolios
- Efficient portfolios lie on the *efficient frontier*
- The decision of which combination of risk and return to choose along the efficient frontier depends on the investor's trade-off between risk and return

Minimum-Variance Frontier and Efficient Portfolios



The Efficient Frontier

Different combinations of assets in the portfolio can be plotted according to their Risk/Return trade-off



Efficient frontiers

- Some possible combinations of assets are sub-optimal: it is possible to get better risk/return combinations
- By eliminating the sub-optimal points it is possible to construct the *efficient frontier*
- The decision of which combination of risk and return to choose along the efficient frontier depends on the investor's trade-off, or indifference, between risk and return

Finding the Efficient Set

- Required inputs are:
  - expected return
  - variance/standard deviation
  - covariance/correlation
- In practice, a computer is used to perform the numerous mathematical calculations required. Technically, this is quadratic programming problem. It is easily solved using, for example, the *Solver* facility in Excel (Computer workshop!)

Portfolio Input Measures

- Historic data
  - long run data. Issues?
  - performance under different economic conditions: Use sub-period analysis?
  - are past averages a good indicator of the future?
- Scenarios
  - test different future "states"
  - see Matysiak JPF Vol. 3/4 1993 pp68-75
- Forecasts
  - Models / Capture 'Dynamics' / Costs / Accuracy?

PROPERTY & PORTFOLIO THEORY

Nominal returns 1987-2006 (source: Hoesli/Lizieri)

Asset	Return	Risk	Risk / Return
Shares	12.1	15.25	0.79
Gilts	9.4	5.11	1.83
Property	11.1	4.59	2.42

Correlations:

	Shares	Gilts	Property
Shares	1.000	.350	-0.011
Gilts		1.000	-0.15
Property			1.000

The story changes somewhat depending on time period and data series used

Investment question

- Let's say that you decide to invest in a *diversified* equity portfolio with average risk. You obtain a return that was 20%.
    - is this satisfactory?
- Suppose the FTA All-Share Index has produced, for the same period, a total return of 15%.

  Can you say that the fund, for this period, had a superior return?

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